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CONDUCTING-SHEET MODEL FOR NATURAL CONVECTION THROUGH A DENSITY-STRATIFIED INTERFACE

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Abstract—A simple physical model is presented for prediction of heat transfer through the sharp density-stratified interface between two immiscible fluids, one lying atop the other and both in turbulent natural convection. The interface is treated as an infinitely thin, highly conductive, rigid membrane. The resulting prediction is tested against experimental data whose Rayleigh numbers for the individual layers range from 10^7 to 10^{10} and whose Prandtl numbers range from 3·3 to 920. The prediction agrees well with the data.

NOMENCLATURE

g, acceleration of gravity;
k, thermal conductivity;
L, fluid layer depth;
q, heat flow;
T_h, lower surface temperature;
T_c, upper surface temperature;
T_i, temperature of the interface;
A, cross sectional area.

Greek
β, coefficient of thermal expansion;
ν, kinematic viscosity.

Nondimensional parameters
Nu, Nusselt number, $(q/A)_{\text{actual}}/(q/A)_{\text{conduction}}$;
Pr, Prandtl number;
Ra, Rayleigh number.

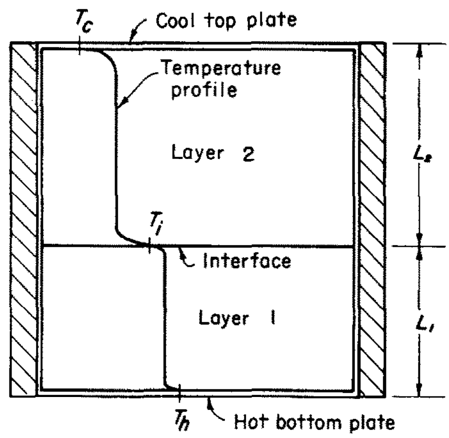

FIG. 1. Nomenclature of dual-layer system.

INTRODUCTION

THE PROBLEM considered arises in connection with analytical [1–3] and experimental [4] studies of thermohaline natural convection, in which salt and heat (or other doubly-diffusive combinations) are allowed to flow simultaneously under bouyancy forces between horizontal planes. Under appropriate conditions thermoclines may be formed, and convection experiments which include the influence of thermoclines form an attractive field of study. In the early stages of this work, it was not considered a certainty that the desired thermoclines could be created in the laboratory; yet it seemed essential to provide some simple and readily reproducible form of stratified natural-convection system as soon as possible. One possibility, the one discussed here, is to allow convection between two immiscible liquids which stratify statically because one is lighter than the other. Under this arrangement the only transported quantity is heat, which is supplied from the bottom, transferred by turbulent natural convection through the two layers of immiscible fluid, and removed at the top in steady state (Fig. 1).

Normal-mode perturbation analyses have been performed for a system with a sharp, density-stratified interface between two immiscible fluids. Welander [5] considered the thermal stability of such an interfacial system with heating from above. Although this configuration is usually stable, Welander stated that for a certain value of a property ratio the system could become unstable. He suggested experiments to test this theory (water above mercury), but no data were presented and the theory remains unsubstantiated. Ruehle [6] considered a similar system, but with heating from below. A stability analysis was performed considering the interface between two immiscible fluids to be a hydrodynamically rigid surface, that is, having zero normal and tangential velocity at the interface. Ruehle concluded that the critical Rayleigh number for onset of convection in one of the two fluids is reduced below the "fixed-fixed" value for a single fluid but that otherwise the interface is like a rigid surface. He substantiated his findings with experiments which focused on initial instability and the early stages of laminar natural convection.

There has been little work, however, in the study of turbulent natural convection through a density-stratified interface, particularly to predict or measure the value of the heat flux. Prenger [7] performed a study of turbulent natural-convection heat transfer between immiscible liquids, but his attempt to develop a suitable set of correlating parameters with which to predict the heat flux was unsuccessful. It was the purpose of this study, then, to obtain a relation by which the turbulent natural convection heat transfer through a sharp density-stratified interface could be predicted.

PHYSICAL MODEL

The system consisting of two immiscible fluids superposed one on the other in a density-stable configuration is heated from below and cooled from above. As in Ruehle's study [6], the interface between the two fluids is modeled as a very thin rigid surface, across which there is no temperature drop. It is seen that with the "conducting sheet" between layers the heat transfer through each fluid layer reduces to classic, turbulent natural convection between parallel flat plates with heating from below. The nondimensional turbulent heat flux through each singe fluid layer may be expressed as

$$Nu_1 = CRa_1^m Pr_1^n \quad \text{and} \quad Nu_2 = CRa_2^m Pr_2^n, \tag{1}$$

where the subscripts 1 and 2 refer to the lower and upper layer respectively.

The coefficients C, m and n must be determined experimentally. There are analytical predictions of these constants, but they are successful only to a qualitative degree. With the first law of thermodynamics and equation (1), the heat flux relation for the layered system may now be developed.

HEAT-FLUX RELATIONSHIP

The steady-state heat balance gives $(q/A)_1 = (q/A)_2$. Each of these fluxes may be computed from equation (1) to give

$$(q/A)_1 = C\frac{k_1}{L_1}\left(\frac{g\beta_1}{v_1^2} L_1^3 Pr_1^{1+n/m}\right)^m (T_h - T_i)^{1+m}$$

and

$$(q/A)_2 = C\frac{k_2}{L_2}\left(\frac{g\beta_2}{v_2} L_2^3 Pr_2^{1+n/m}\right)^m (T_i - T_c)^{1+m},$$

which, together with the identity

$$T_h - T_c = (T_h - T_i) + (T_i - T_c),$$

may be used to eliminate the unknown interface temperature T_i. The heat flux is then

$$q/A = \frac{C\{\alpha_1\alpha_2\}^m (T_h - T_c)^{1+m}}{[(\alpha_1)^{m/(1+m)} + (\alpha_2)^{m/(1+m)}]^{1+m}} \tag{3}$$

where for each layer

$$\alpha = \alpha = \left(\frac{k}{L}\right)^{1/m} \frac{g\beta}{v^2} L^3\, Pr^{1+n/m}.$$

The conduction solution, the solution for q/A if all heat transport were by conduction, is

$$(q/A)_c = \frac{(T_h - T_c)}{\frac{L_1}{k_1} + \frac{L_2}{k_2}}.$$

As the Nusselt number is defined as the ratio of the actual heat transfer to the heat transfer by conduction alone, its value is the ratio of the previous two quantities, or

$$Nu_i = C\left\{\frac{\left[\frac{L_1}{k_1} + \frac{L_2}{k_2}\right]^{1/m} \alpha_1\alpha_2(T_h - T_c)}{[(\alpha_1)^{m/(1+m)} + (\alpha_2)^{m/(1+m)}]^{(m+1)/m}}\right\}^m. \tag{4}$$

The term inside the braces is of similar form as the ordinary Rayleigh number for a single layer system, except that the additional Prandtl number dependence Pr^m is now included by way of α_1 and α_2. This expression will be referred to as the "modified interfacial Rayleigh number", Ra_{mod_i}, leaving

$$Nu_i = CRa_{\text{mod}_i}^m.$$

As is discussed in the next section, an experiment was performed to determine the constants C, m and n for turbulent natural convection in a single fluid layer. The constants were found to be $C = 0{\cdot}0535$, $m = 0{\cdot}333$, and $n = 0{\cdot}084$. The prediction of the nondimensional heat flux through a density-stratified interface then becomes

$$Nu_i = 0{\cdot}0535\, Ra_{\text{mod}_i}^{0{\cdot}333}. \tag{5}$$

This is the equation to be tested against experiments executed for the immiscible, density-stratified liquids water and silicone oil; it is the result of the "conducting-sheet" model of interfacial convection.

EXPERIMENTAL CONFIRMATION

Two groups of experiments were executed, a series of three preliminary experiments whose purpose was to establish the validity of the main experiment, and the experimental confirmation of equation (5). The first set of preliminary experiments was done for a single layer of liquid in the apparatus eventually used for the double-layer tests. The object was to determine the constants C, m and n of equation (1) in view of the well known tendency of these quantities, especially C, to depend (to $\pm$ 100 per cent) on the particular tank used.

This first preliminary experiment was conducted with water, 1·0 cSt silicone oil, 10·0 cSt silicone oil in cylindrical test sections of $8\frac{1}{4}$ in. dia. and $1\frac{3}{4}$, 3 and $4\frac{1}{2}$ in. depths. The results of these tests, plotted in Fig. 2, are

$$Nu = 0{\cdot}0535\, Ra^{0{\cdot}333}\, Pr^{0{\cdot}084} \qquad (6)$$

for $10^6 < Ra < 10^9$ and $5{\cdot}25 < Pr < 83$. These results are in reasonable agreement with the values commonly found in the literature [8].

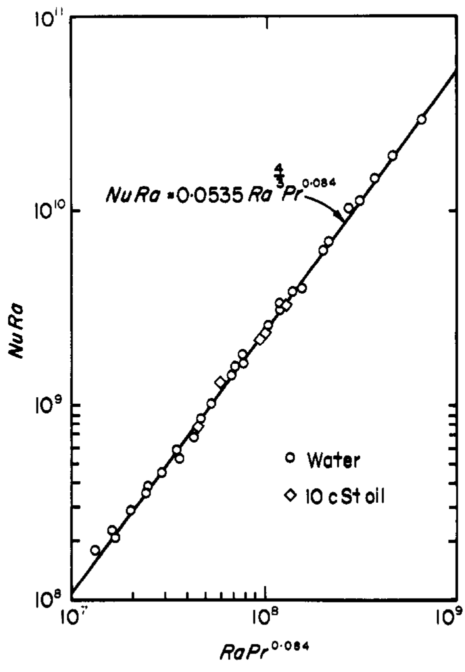

FIG. 2. Data for single-layer convection with resulting Nusselt–Rayleigh correlation.

The second preliminary investigation was to determine if the presence of the lateral walls inhibited convection for various depth-to-diameter ratios in the test sections used in the first preliminary set of experiments. Experiments were run using water with depth-to-diameter ratios from 0·125 to 1·815 for Rayleigh numbers from 5×10^5 to 10^9. It was found that in this experiment there was no inhibition of convection due to the lateral walls. These findings are in good agreement with the conclusions of Catton and Edwards [9], who found that a depth-to-diameter ratio greater than 2·5 was needed in order to inhibit convection significantly at Rayleigh numbers around 10^6. Thus the results of the first preliminary study may be considered applicable for the geometries considered, and they are not biased by lateral wall inhibition.

The third set of data was taken with substantially the same test section as was used in the main experiment. In this experiment, a 5-mil-thick aluminum sheet was placed horizintally in the test section. Data were taken with water above and below the sheet, as well as with 1 cSt silicone oil above and water below the sheet. The measured heat flux was correlated with the modified interfacial Rayleigh number as suggested by the derived heat flux relation. A plot of the data and the predicted curve is given in Fig. 3. Equation (5) passes through the data in the case where there is actually a conducting sheet present. A benchmark has been established against which the main experiment can be compared.

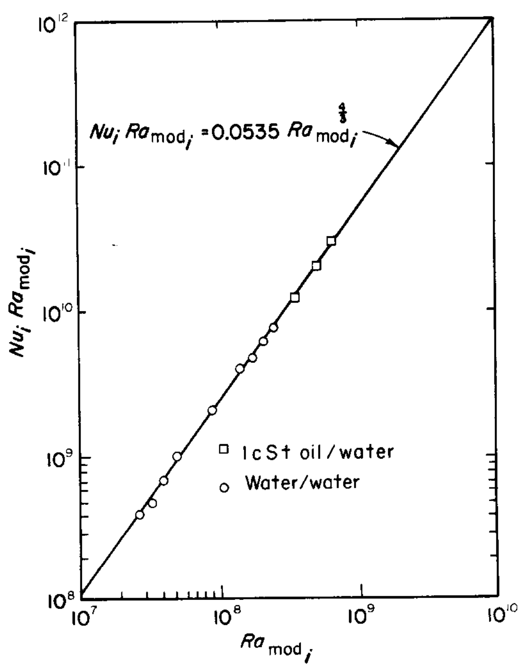

FIG. 3. Data for convection with thin aluminum plate separating the layers.

The confirmation of equation (5) was executed in a circular plexiglas tank of $8\frac{1}{2}$ in. dia and variable depth. Heat was supplied at the bottom by an electric heater, and the top cooling was obtained with a cooling-water coil. Suitable precautions were taken to minimize heat losses to the sides. Thermocouples were provided to measure the bulk temperatures in the two layers, the top and bottom wall temperatures, and the temperature distribution in the side-wall insulation.

Data were taken and analyzed for systems of Dow-Corning No. 200 and 210H silicone oils of 1, 10 and 100 cSt viscosity, each with distilled, degassed water as the lower fluid. The raw data are presented in [8]. Preliminary reduction showed the individual fluids to be in the turbulent regime with individual-layer Rayleigh numbers ranging from 10^7 to 10^{10}. The data were then reduced in terms of interfacial Nusselt numbers and modified interfacial Rayleigh numbers.

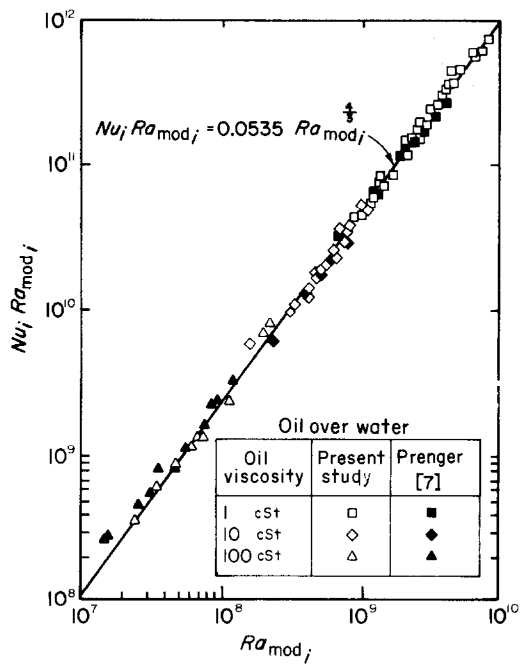

FIG. 4. Data for convection between two immiscible liquids, including prediction equation.

The plot of these data and the curve formed by equation (5), in Fig. 4, are seen to be in close agreement. The earlier data taken by Prenger [7] for a two-layer system of immiscible fluids were reduced in accordance with the physical model of this work, and again good agreement was found. Prenger's data, correlated by this model, are included in Fig. 4.

CONCLUSIONS

A physical model has been developed to describe the heat flux of turbulent natural convection through a density-stratified interface between immiscible fluids.

Data from experiments as well as from other investigations have been found to verify this prediction of heat flux. The predicted and experimental Nusselt numbers are expressed by equation (5).

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MODÈLE DE COUCHE CONDUCTRICE POUR LA CONVECTION NATURELLE A UNE INTERFACE

Résumé—On présente un modèle simple pour le calcul de transfert thermique à travers l'interface avec forte stratification de densité entre deux liquides non miscibles, l'un surmontant l'autre et tous les deux en état de convection naturelle turbulente. L'interface est considérée comme une membrane très conductrice et rigide.

Les résultats du calcul sont confrontés à l'expérience faite pour des nombres de Rayleigh compris entre 10^7 et 10^{10} et des nombres de Prandtl entre 3,3 et 920 pour chaque couche. Le calcul s'accorde bien avec les résultats des mesures.

SCHEIBENMODELL DER WÄRMELEITUNG FÜR DIE NATÜRLICHE KONVEKTION DURCH EINE DICHTEGESCHICHTETE GRENZSCHICHT

Zusammenfassung—Es wird ein einfaches physikalisches Modell angegeben, das eine Voraussage über den Wärmedurchgang durch eine Grenzschicht mit Dichteschichtung zwischen zwei nicht-mischbaren Flüssigkeiten gestattet, die übereinander liegen und sich in turbulenter natürlicher Konvektion befinden.

Die Grenzschicht wird als unendlich dünn, höchst wärmeleitend und stabil betrachtet. Die resultierende Voraussage wird mit experimentellen Werten verglichen, deren Rayleighzahlen für die einzelnen Schichten zwischen 10^7 and 10^{10} und deren Prandtl-Zahlen zwischen 3,3 und 930 liegen. Die Voraussage stimmt gut mit den Messwerten überein.

МОДЕЛЬ ПРОВОДЯЩЕГО СЛОЯ ПРИ ЕСТЕСТВЕННОЙ КОНВЕКЦИИ ЧЕРЕЗ ПОВЕРХНОСТЬ РАЗДЕЛА СО СКАЧКОМ ПЛОТНОСТИ

Аннотация—В работе представлена простая физическая модель для расчета теплообмена через поверхность раздела со скачком плотности между двумя несмешивающимися жидкостями, находящимися одна над другой в условиях турбулентной естественной конвекции. Поверхность раздела представляет собой бесконечно тонкую жесткую мембрану с высокой теплопроводностью.

Результаты расчета сопоставлены с экспериментальными данными, полученными в диапазоне чисел Релея для отдельных слоев от 10^7 до 10^{10} и чисел Прандтля от 3,3 до 920. Расчет хорошо согласуется с экспериментальными данными.